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FLOW IN SMOOTH STRAIGHT PIPES AT VELOCITIES ABOVE AND BELOW SOUND VELOCITY

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FLOW IN SMOOTH STRAIGHT PIPES AT VELOCITIES ABOVE AND BELOW SOUND VELOCITY*

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To investigate the laws of flow of compressible fluids in pipes, tests were carried out with air flowing at velocities below and above that of sound in straight smooth pipes. The transition from supersonic to subsonic velocity is particularly noteworthy.

The laws of flow in smooth pipes have been thoroughly investigated with regard to flow resistance and velocity distribution for incompressible fluids (reference 1). These laws are applicable to liquids and gases, provided the differences in density set up may be neglected. For air this will generally be the case for velocities up to 50 m/s (100 mi./hr.).

The object of the tests described below is the investigation of the laws of pipe flow in the case of compressible fluids and comparison with the laws for the incompressible case. Air was chosen as the flow medium. In order that the effect of the compressibility may be brought out most effectively, the velocity should lie between 100 and 500 m/s (200 and 1,000 mi./hr.); that is, be of the order of magnitude of the velocity of sound in the air. The best series of tests that have so far been recorded in the literature on the subject are those described by G. Zeuner (reference 2), who has also carefully developed the theory of the phenomenon for velocities below that of sound.

The computational treatment of this type of flow belongs to the field of application of the "Dynamics of

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*"Strömung in glatten, geraden Rohren mit Über- und Unterschallgeschwindigkeit." Forschung auf dem Gebiete des Ingenieurwesens, March-April 1936, pp. 75-84.
Tests carried out at the Kaiser-Wilhelm Institute for Flow Investigation.
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Gases" where, in addition to the velocity w (m/s) and the pressure p (kg/m²), the density $\rho = \gamma/g$ (kg s²/m⁴) and the temperature T (°K) are also variable. The computations can be carried out without undue difficulty only under definite assumptions. The most important assumption is that the energy of the gas remains constant during the flow process - i.e., that no heat be added or conducted away. It is therefore necessary that the flow channel be well heat insulated. The tests brought out the fact that as a result of friction, the gas temperature in the neighborhood of the wall acquires approximately the temperature of the outside air, so that no heat exchange takes place through the pipe wall.

The setting up of a velocity above that of sound in cylindrical pipes is made possible only through the application of special means. (See section 12.) An important part is here also played by the "compression shock" (or shock wave) which converts the supersonic into a subsonic velocity.

I. TESTS

1. Tests set up.- The test set up (fig. 1) consists of two vacuum tanks a, with a total volume of 20 m³, a gasometer or gas reservoir b, of 25 m³ capacity, a vacuum pump, c, (380 m³/hr., 10 hp.), an air-drying apparatus d, filled with calcium chloride, a quick-closing cock e, (described by Nikuradse (reference 3)) with corresponding accessories, and the test portion f.

The test procedure is divided into a preliminary part and the test proper. In the preliminary portion of the test the vacuum pump c sucks the air out of the vacuum tanks a and delivers it through the drying apparatus d to the gasometer b, in which the air is at approximately atmospheric pressure. During this process the shut-off member e between the test section and the vacuum tanks is closed. The portion of the tests in which measurements are made, starts with the opening of the shut-off member e. As a result of the above atmospheric pressure, the air streams through the test length into the vacuum tanks a and gradually fills them. On account of the presence of the velocity of sound in a portion of the test length, the condition of the flow in f remains strictly constant until a certain pressure is attained in the vacuum tanks The duration of the flow as compared with that of the usual wind tunnel is therefore limited, depending chiefly on the capacity of the tanks and on the minimum flow cross section. The test intervals are of the order of magnitude of 10 to 100 seconds. On account of those short test intervals, it is necessary that the opening times of the shut-off member e be very small (about 0.2 to 0.1 sec.). If the constant flow in the test portion f discontinues before the test has been brought to an end, the vacuum tanks may again be emptied, with the shut-off cock closed and the test repeated in the same manner. The same flow conditions are again established if the initial flow condition (pressure and temperature at the inlet to the test section) remains as before. Here is where the great advantage of working with a set-up at below atmospheric pressure as compared with one at above atmospheric pressure occurs, namely, that in the first case the initial state - being that of the atmosphere - is constant for several succeeding tests, whereas in the second case the initial state varies even during a single test and must constantly be observed. Since the moisture contained in normal air freezes at the low temperatures of the air stream and would give rise to disturbances, it is necessary that the air be dried.

2. Test portion.- The test portion f consists of a simple, smooth cylindrical pipe with nozzles placed at either end. At certain intervals along the pipe, static pressure orifices are located at which the pressure is transmitted through tubes to the manometers. It is necessary to distinguish three kinds of pipe arrangements:

a) For velocities below that of sound (fig. 2): The pipe receives a well-rounded inlet of the same size as the inner pipe diameter. At the end of the pipe is connected a velocity-measuring apparatus g with a dynamic pressure tube displaceable in three directions. The tip of the tube projects 2 mm into the test pipe, in which is also contained a static-pressure orifice in the pipe wall. Behind this apparatus there is a nozzle whose minimum cross section is smaller than that of the pipe section, so that the flow velocity in the test portion settles down to a definite value. To each velocity of flow there corresponds a definite nozzle.

b) For velocities above that of sound (fig. 3): In this case the nozzle is placed ahead of the pipe entrance. The minimum cross section of the nozzle is again smaller than that of the pipe section and in this case also regulates the velocity of the flow in the test portion. The velocity distribution is again measured by a dynamic pressure tube at the end of the pipe.

c) For the velocity of sound (fig. 4): The pipe receives a well-rounded inlet at the entrance and at the outlet a nozzle whose minimum cross section is larger than that of the pipe cross section, so that the pipe offers the only appreciable resistance and the velocity of flow in the test section is self-regulating. In this case there is also one flow condition characterized by the fact that at the end cross section of the pipe the mean velocity of the flow is equal to that corresponding to the velocity of sound at the local gas state.

3. Pressure measurement.- To measure the pressures, simple U manometers of glass were employed, set up on a stand next to one another. One leg of a U tube communicated through a tube connection with a corresponding measuring station at the pipe, the other communicating with the atmosphere, so that the difference between atmospheric pressure and the pressure to be measured was indicated. The real (absolute) pressure is the sum of the pressure difference read off and the barometer pressure. The manometer tubes were filled with mercury. On account of the small duration of the tests, the readings were obtained by using indices that were easily displaceable on the glass tubes and read off on a scale only at the completion of the test.

4. Measurement of the discharge.- The quantity flowing through per second G (kg/s) was computed from the displacement velocity c_G (m/s), the section F_G (m^2) of the bell of the gasometer b and the specific weight γ_G (kg/m³) of the air in the bell, according to the equation $G = F_G \, c_G \, \gamma_G$. The bell cross section was found to be 10.74 m^2. The velocity at the bell could be observed with the aid of a make-and-break device and a stop watch at the test stand. The specific weight γ_G of the air was computed from the gas equation, taking into account the temperature T_G (°K) of the air, and the pressure p_k (kg/m²) in the bell. The temperature of the air in the bell could not be determined with the ordinary mercury thermometer on account of the poor accessibility. Moreover, the large inertia of such a thermometer would not permit an accurate indication of the temperature in the short test interval available. There was there-

fore measured the change in the electrical resistance of a platinum wire and this change converted into temperature readings by a previous calibration. For the resistance, a platinum wire of 0.1 mm diameter and 3.75 m long, and having a resistance of about 50 ohms, was used, the wire being wound on an insulating frame which was also a poor heat conductor. The frame was mounted in the downtake tube of the gasometer (fig. 1), so that the air flowing out from the gasometer streamed past it. As a result of the good contact of the air with the wire and the small specific heat of the wire, the temperature was soon brought to a steady value. A device inside the bell provided for a good mixing of the air before the test so that its temperature was sufficiently uniform. The resistance of the platinum wire was measured with a Wheatstone bridge whose range made possible the measurement of a temperature difference of 40°. One degree temperature difference corresponded on the scale to about 1 cm length, so that the accuracy in reading, using a very sensitive galvanometer (one division = 0.00001 V), was to about 0.1° and was quite sufficient for computations with absolute temperatures.

5. <u>Test procedure</u>.- The tests proper were preceded by a number of preliminary tests. The object of the latter was chiefly to investigate the effect produced on the flow by the insertion of the stem of the pitot tube. It turned out that at velocities below that of sound, an appreciable disturbance in the flow occurred, whereas at velocities above that of sound no disturbance was observed. Since every disturbance is propagated with the speed of sound, it was possible for the disturbance to travel upstream in the first case, while in the second case it was driven downstream. The disturbance was eliminated by providing the pitot tube with a fixed covering and moving the tube vertically only (fig. 5). The covering of course produced a certain throttling of the stream - which throttling, however, was constant for each position of the tube and led only to an increase in the normal desirable throttling through the presence of the nozzles.

With velocities below that of sound the pipe could be of arbitrary length. The effect shows up only in the lowering of the quantity discharged as a result of the increase in the wall friction for equal drop in pressure. Velocities above that of sound, however, could only occur for definite pipe lengths. In this case the wall friction has a slowing-down effect on the air velocity leading to a pressure rise. If the wall friction is too great, i.e., the pipe too long, there is a great deceleration of the flow and hence a great pressure rise which, beyond a certain limiting length, ends up in a compression shock. The latter converts all supersonic velocities into subsonic velocities and may occur at any position in the pipe, depending on the total wall friction. As the friction increases the compression shock travels upstream until the entrance of the pipe is reached, so that beyond a certain pipe length no supersonic flow is any longer possible. The result may be obtained if throttling is substituted for the friction. With supersonic velocities, care is to be particularly taken that the nozzle at the pipe entrance is well shaped, since it produces an additional harmful frictional resistance which increases as the length of the nozzle increases. In order to reduce this frictional resistance the nozzle should be kept as short as possible. In this connection there is, however, to be considered the fact, characteristic of supersonic flow, that too sudden divergences in the stream give rise to periodic fluctuations (waves) which strongly affect the pressures measured at the wall. In order to prevent the occurrence of these waves the divergence must increase gradually; i.e., the nozzle must be long. Both effects therefore oppose each other. Only by special tests is it possible to determine the most suitable form of nozzle having the least effect on the flow.

II. EVALUATION OF THE TEST RESULTS

1. General.- The measurements were carried out on four pipes with diameters $d = 10$, 20, 25, and 30 mm. The length l of the pipe was each time so chosen that the following l/d ratios were obtained.

a) At velocities below that of sound: $l/d = 360$, 324, 288, 252, 216, 180, 144, 108, 72, 54, 36, 18, 12, 6, 3, and 0.32.

b) At velocities above that of sound: $l/d = 30$, 26, 16, and 12. The pipes were provided with manometer connections. In figure 2, x denotes the distance measured from any orifice position to the pipe entrance.

The following measurements were taken on all the pipes:

1. Pressure drop or rise along the pipe measured at the pressure orifices.

2. Quantity of air flowing through per second measured from the displacement velocity at the gasometer bell, the air temperature and the pressure in the bell.

3. Air temperature just ahead of the entrance to the test pipe measured with a mercury thermometer.

4. Initial pressure (barometer reading + the pressure above or below atmospheric in the bell).

5. Moisture of the air measured with a psychrometer just ahead of the entrance to the test pipe in order to test the efficiency of the drying apparatus.

6. Velocity distribution over the pipe cross section. These measurements were carried out only for the pipes of 25 mm diameter.

2. Relation between pipe length, pressure, and rate of discharge.- From the pressures p along the pipe, and the initial pressure p_k (barometer reading + pressure in bell) the ratio p/p_k was formed, and similarly, the ratio $G/f = \Psi$ from the quantity weight flowing through per second G, and the corresponding cross-sectional area f, and the ratio x/d from the distance x of any orifice to the pipe entrance and the inner diameter d of the pipe. The values of Ψ were made nondimensional by dividing by Ψ_{crit}, whose meaning will be further explained below (equation 2b). Plotting p/p_k against x/d, the curves of figure 6 are obtained for velocities below that of sound. It will be seen that the curves drop slowly at first, then more rapidly. Each curve corresponds to a value of the ratio Ψ/Ψ_{crit} proportional to the rate of discharge. The lower ends of all the curves are joined by a dotted line which tends to approach $p/p_k = 0$ as x/d approaches infinity. This curve gives the limiting value of the pressure ratio p/p_k for given lengths l/d and clearly shows that the increase in the pressure drop for increasing pipe length is very considerable for short pipes, whereas for long pipes there is only a small increase. For example, for an increase in length from $x/d = 0$ to 185, the pressure difference p/p_k lies between 0.527 and 0.277; that is, a difference of 0.25. The same difference is also obtained for $x/d = 270$ to infinity, in which case p/p_k lies between 0.25 and 0.

For velocities above that of sound the curves rise slowly at first, some of them showing a sudden pressure rise, after which they again drop in the same manner as the curves of figure 6. Figure 7 shows the curves for the pipe of 25 mm diameter. The sudden pressure rise indicates a compression shock which arises only in the case of velocities above that of sound if the quantity flowing through for a given pipe length, or conversely, the pipe length for a given quantity flowing through becomes too large. For this given rate of discharge the pipe, therefore, has too much throttling, so that in the upstream portion of the pipe, only subsonic velocities can occur. Here, too, each curve is marked by the corresponding value ψ/ψ_{crit}.

Both sets of curves were employed to plot p/p_k against ψ/ψ_{crit}, where x/d now appears as the parameter (fig. 8). To plot these curves, the values of p/p_k for definite values of x/d at constant ψ/ψ_{crit} were taken from figures 6 and 7. Vertical sets of points were thus obtained (fig. 8), representing the pressure variation along a definite pipe. The numbers above the curves give the corresponding diameters of the pipes for which the series of points was obtained and show that the absolute measure of the diameters has no effect on the trend of the curves but only the ratio l/d or x/d. Each series of points also shows clearly that the various pipe diameters give the same values for equal ratios x/d or l/d. All symbols of the same kind denote equal ratios of x/d or l/d, and are joined to one another by curves. For all series of points which do not extend to the dotted line, throttling was obtained by placing nozzles behind the pipe (fig. 2). Those points also show good agreement with the others and it may be seen that throttling is equivalent to the frictional resistance of a given increase in pipe length. In order to determine the pipe-length increase corresponding to the throttling, it is only necessary to obtain from figure 9 the total pipe length for the corresponding value of ψ/ψ_{crit}, which in this case becomes ψ_{max}/ψ_{crit}, and form the difference. Figure 9 was obtained by taking from figure 6 the maximum quantities flowing through for each given length of pipe l/d. The continuous curve corresponds to the empirical relation:

$$\psi_{max}/\psi_{crit} = 0.916\left\{(l/10d)^{0.819}\right\} \qquad (1)$$

Figure 8 shows clearly the relations between the values x/d, p/p_k, and ψ/ψ_{crit} which are all interconnected for any pipe diameter; i.e., to a given value of x/d and ψ/ψ_{crit} there corresponds a definite ratio p/p_k and conversely. The entire series of curves, which includes all flow conditions in smooth, straight pipes, splits up into two sets: a downward sloping set of curves for the velocities below that of sound, and an upward sloping set of curves for the velocities above that of sound. The boundary between the two sets is represented by a dotted straight line which extends from the origin to the critical point, the significance of the latter being explained below. The curve denoted by $x/d = 0$, which envelops the entire series, gives the theoretical discharge rate for a pipe-length zero (zero friction) and a given pressure ratio p/p_k. For this curve, the relation

$$\psi_{theor} = w/v \tag{2}$$

holds true where v (m^3/kg) is the specific volume. The velocity w for the case of the adiabatic process which is here assumed $p\,v^{\kappa} = p_k\,v^{\kappa}_k$ is

$$w = \sqrt{\frac{2g}{\kappa - 1}\,\frac{\kappa}{}\,p_k\,v_k\left[1 - \left(\frac{p}{p_k}\right)^{(\kappa-1)/\kappa}\right]} \tag{3}$$

and substituting in equation (2), there is obtained

$$\psi_{theor} = \sqrt{\frac{2g\,\kappa}{\kappa - 1}\,\frac{p_k}{v_k}\left[\left(\frac{p}{p_k}\right)^{2/\kappa} - \left(\frac{p}{p_k}\right)^{(\kappa+1)/\kappa}\right]} \tag{2a}$$

The theoretically maximum discharge rate is obtained by substituting the critical pressure ratio

$$\frac{p}{p_k} = \left(\frac{2}{\kappa + 1}\right)^{\kappa/(\kappa-1)} \tag{4}$$

We then have:

$$\psi_{crit} = \sqrt{\frac{2g\kappa}{\kappa-1}\,\frac{p_k}{v_k}\left[\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)} - \left(\frac{2}{\kappa-1}\right)^{(\kappa+1)/(\kappa-1)}\right]} \tag{2b}$$

(for $\kappa = 1.405$, $\psi_{crit} = 2.144\sqrt{p_k/v_k}$);

k = stagnation

The enveloping curve is thus given by

$$\frac{\psi_{theor}}{\psi_{crit}} = \sqrt{\frac{\left(\frac{p}{p_k}\right)^{2/\kappa} - \left(\frac{p}{p_k}\right)^{(\kappa+1)/\kappa}}{\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)} - \left(\frac{2}{\kappa+1}\right)^{(\kappa+1)/(\kappa-1)}}} \tag{5}$$

and also contains the critical point which gives the maximum theoretically possible discharge through a cylindrical pipe. For $\kappa = 1.405$, the values have been collected and are given in table I.

TABLE I. ψ_{theor}/ψ_{crit} as Function of p/p_k

According to Equation (5)

p/p_k	ψ_{theor}/ψ_{crit}	p/p_k	ψ_{theor}/ψ_{crit}
1.00	0.0000	0.60	0.9820
.99	.1960	.56	.9995
.98	.2880	.527	1.0000
.97	.3540	.50	.9950
.96	.4155	.45	.9860
.95	.4490	.40	.9650
.94	.4880	.35	.9340
.92	.5570	.30	.8820
.90	.6150	.25	.8180
.88	.6700	.20	.7410
.86	.7150	.15	.6470
.84	.7545	.10	.5210
.82	.7890	.08	.4580
.80	.8170	.06	.3880
.76	.8750	.04	.3040
.72	.9160	.02	.1918
.68	.9530	.01	.1245
.64	.9745	.005	.0783

The subsonic region thus consists of the curves sloping downward from left to right, limited above by half of the theoretical curve, and below by the dotted straight line. All curves start at the point $p/p_k = 1$ and $\psi/\psi_{crit} = 0$. As the length x/d increases, the curves become steeper and the values ψ/ψ_{crit} smaller. The latter approach the limiting value zero as the length approaches infinity.

The dotted straight line results from the consideration that the mean velocity at the end section of the pipe is equal to the velocity of sound in case the pipe forms the only resistance. This may be shown by the following reasoning. The gas flows through a cylindrical pipe and suffers a pressure drop. This results first in an increase in velocity, according to equation (3), and second in a lowering of the density; both effects depend on the pressure. According to equation (2), however, there is obtained from the density and velocity, the discharge Ψ, which has the same value over the entire pipe length on account of the constant cross section, so that both magnitudes are interconnected. Below the velocity of sound, at zero friction and with pressure drop, the velocity increases more rapidly than the density decreases so that the quantity that has entered with friction, can flow through each cross section along the pipe with additional pressure drop. Above the velocity of sound, however, without friction and with decrease in pressure, the density decreases faster than the velocity increases. Without a rise in pressure the mass of air cannot therefore flow along the pipe through every section. A rise in pressure in the cylindrical pipe is possible, however, only with a previous pressure drop obtained through nozzles at the pipe entrance. Since this is not the case with the pipe arrangement here employed, this condition and hence the supersonic flow, is not possible. The limit lies exactly at the velocity of sound a, so that this condition may still be reached and is established at the end section of a cylindrical pipe.

For the end section, we therefore have the relation:

$$\overline{w} = \overline{a} = \sqrt{\kappa\ g\ R\ \overline{T}} = \sqrt{\frac{2g\kappa}{\kappa-1}\ R\ \overline{T}\left(\frac{T_k}{\overline{T}} - 1\right)} \qquad (6)$$

where mean values are indicated by strokes above the letters. From the above is obtained the equation $\overline{T}/T_k = 2/(\kappa+1)$ with the equation of state:

$$\frac{p\overline{v}}{p_k\ v_k} = \frac{2}{\kappa + 1} \qquad (7)$$

Substituting in equation (2) for w, the velocity of sound according to equation (6) and squaring, there is obtained:

$$\psi^2 = \kappa \, g \, p/\bar{v} \tag{8}$$

From (7) and (8), there is finally obtained:

$$\left(\frac{p}{p_k}\right)^2 = \psi^2 \, \frac{v_k}{p_k} \, \frac{2}{g \, \kappa \, (\kappa+1)} \tag{9}$$

The right side of (9) together with equation (2) gives the dotted straight line:

$$\left(\frac{p}{p_k}\right)^2 = \left(\frac{\psi}{\psi_{crit}}\right)^2 \frac{4}{\kappa^2 - 1} \left[\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)} - \left(\frac{2}{\kappa+1}\right)^{(\kappa+1)/(\kappa-1)}\right] \tag{10}$$

For $\kappa = 1.405$, there is obtained $p/p_k = 0.527 \; \psi/\psi_{crit}$, and for $\psi/\psi_{crit} = 1$, i.e., for the velocity of sound in the short nozzle (without friction), $p/p_k = 0.527$.

In the above relation it is assumed that the velocity distribution in the end section of the pipe is rectangular, i.e., that the velocity is equal to the sound velocity over the entire cross section. Actually, however, a different velocity distribution exists, resulting in a change in the slope of the line. The latter is shown as the continuous line in figure 8. For this line, according to the measured values of $\psi/\psi_{crit} = 0$ to 0.855 there holds the relation:

$$\frac{p}{p_k} = \frac{\psi}{\psi_{crit}} \, 0.911 \sqrt{\frac{4}{\kappa^2 - 1} \left[\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)} - \left(\frac{2}{\kappa+1}\right)^{(\kappa+1)/(\kappa-1)}\right]} \tag{11}$$

The new line meets the theoretical straight line at both end points, and hence must have a short curvature at some point. This point lies approximately at $\psi/\psi_{crit} = 0.855$ and corresponds approximately to the value $x/d = 36$. This means that below $\psi/\psi_{crit} = 0.855$ and above $x/d = 36$, the velocity distribution does not change with increasing pipe length. Above $\psi/\psi_{crit} = 0.855$ and below $x/d = 36$ the velocity distribution with decreasing pipe length gradually approaches the rectangular distribution; that is, approaches the assumption made for equation (10), so that the continuous line also approaches more closely the theoretical straight line and meets it at the critical point. This result is confirmed in figure 11, according to which, beyond the value $x/d = 36$, all velocity distributions coincide.

From equations (1) and (11) the pressure ratio for the maximum discharge for given lengths of pipe may also be computed. For $\kappa = 1.405$, this ratio is:

$$p/p_k = 0.4801 \times 0.916^{\left\{(l/10\ d)^{0.619}\right\}} \tag{12}$$

The region for velocities above that of sound is limited above by the boundary line just described, and below by the second half of the enveloping line. All measured values have again been plotted in figure 8 as vertical series of points, symbols of the same kind denoting equal pipe lengths l/d and being joined together by curves. The numbers indicated on the figure give the corresponding lengths of pipe. It may be seen that the supersonic region is much smaller than the subsonic region and is already practically filled out at the smaller pipe lengths since measurements on greater pipe lengths are made difficult by the very small pressures encountered. Although the two regions approach each other closely at the boundary, no gradual transition between the two sets of curves is to be expected. On the contrary, the transition occurs discontinuously through a compression shock.

All pipe-flow conditions for smooth, straight pipes are thus completely represented by the diagram and any other flow may be determined from it. For example, for a ratio $\Psi/\Psi_{crit} = 0.5$ attained in any cylindrical pipe, a vertical line A-A is drawn through the curves of figure 8. It may be seen that up to a pipe length of about $l/d = 20$, flow is possible with velocities above and below that of sound but with the difference that the pressure ratio p/p_k at the supersonic velocity must be smaller than at the subsonic velocity. The gas (in a nozzle) must therefore expand to a much greater extent, whereby the high velocity is attained. At the same time the density and temperature drop very sharply. With greater pipe lengths, a compression shock occurs in the case of the supersonic flow. These flow relations are possible for all pipes with the intersected values of x/d, which are then to be taken as l/d. The corresponding pressure drop p/p_k is different for each length l/d and may be read off at the corresponding point of intersection with A-A. The pressure drop along the pipe may also be determined on the line A-A from the values of x/d below the pipe length. The maximum pipe length for $\Psi/\Psi_{crit} = 0.5$ is $l/d = 280$. For all greater lengths the air does not flow through. This means that to each pipe length there corresponds a maximum discharge quantity, and the latter may be obtained from the curve already mentioned (fig. 9).

3. Law of resistance.- In order to be able to make comparisons with the friction tests carried out with water (reference 3), the friction values were here also computed from the pressure drops along the pipe. All magnitudes are average values. The resistance coefficient λ is defined in the reference cited as

$$\lambda = (d/\overline{q})\ (-\ dp/dx)_r \tag{13}$$

where d is the pipe diameter, $\overline{q}$ the mean dynamic pressure, $(dp/dx)_r$ the pressure drop along the pipe due to the wall friction. The total pressure drop in our case consists of two parts:

1. Pressure drop through acceleration of the gases at velocities below that of sound, or pressure rise through deceleration of the gases at velocities above that of sound, $(dp/dx)_b$;

2. Pressure drop through friction at the wall $(dp/dx)_r$. We thus have:

$$\frac{dp}{dx} = \left(\frac{dp}{dx}\right)_b + \left(\frac{dp}{dx}\right)_r \tag{14}$$

The dynamic pressure $\overline{q} = \overline{\rho}\ \frac{\overline{w}^2}{2}$. Since heat is neither added nor conducted away, $\overline{w}$ may be determined from the energy equation and the equation of continuity:

$$\frac{\overline{w}^2}{2g} = \frac{\kappa}{\kappa - 1}\left(p_k\ v_k - p\ \frac{f}{G}\ \overline{w}\right) \tag{15}$$

Substituting $G/f = \Psi$ and $p_k\ v_k = R\ T_k$, there is obtained

$$\overline{w} = \sqrt{\frac{2g\ \kappa}{\kappa - 1}\ R\ T_k + \left(\frac{g\ \kappa}{\kappa - 1}\ \frac{p}{\Psi}\right)^2} - \frac{g\ \kappa}{\kappa - 1}\ \frac{p}{\Psi} \tag{16}$$

For small velocities this mode of computation is too inaccurate. Setting for briefness:

$$a = \frac{1}{2g\kappa/(\kappa-1)}, \quad b = pf/G = p/\psi \quad \text{and} \quad c = - R\ T_k = - p_k\ v_k$$

there is obtained from equation (15) a quadratic equation whose roots are developed into a binomial series. The solution is then obtained as

$$\bar{w} = - \frac{c}{b} - \frac{c^2}{b^3} a - 2 \frac{c^3}{b^5} a^2 - 5 \frac{c^4}{b^7} a^3 \ldots$$

$$= \frac{RT_k}{p/\psi} - \frac{(RT_k)^2}{(p/\psi)^3} \frac{1}{2g\kappa/(\kappa-1)} + \frac{(RT_k)^3}{(p/\psi)^5} \frac{2}{[2g\kappa/(\kappa-1)]^2} -$$

$$- \frac{(RT_k)^4}{(p/\psi)^7} \frac{5}{[2g\kappa/(\kappa-1)]^3} \tag{17}$$

Since $d/\bar{q} = 2gd/\psi\bar{w}$, there is still needed for computing λ the value of $(dp/dx)_r$, and the latter from equation (14) may be obtained as

$$(dp/dx)_r = dp/dx - (dp/dx)_b = dp/dx + \bar{\rho}\ \bar{w}\ d\bar{w}/dx \tag{18}$$

where dp/dx is found by differentiating the curves of p against x. Since these curves have in general a strong curvature (see fig. 6), the differentiation requires a certain amount of care if large errors are to be avoided.

By differentiating the energy equation $\bar{w}^2/2 = \frac{\kappa}{\kappa-1} (p_k/\rho_k - p/\rho)$ and the continuity equation $\bar{\rho}\ \bar{w} =$ const., there is finally obtained, with the aid of equations (18) and (13):

$$\lambda = \frac{d}{\psi} \frac{2g}{\bar{w}} (- dp/dx) \left(1 - \frac{\kappa}{\kappa - 1} \frac{1}{1 + \frac{\kappa}{\kappa - 1} p/\bar{\rho}\ \bar{w}^2}\right) \tag{19}$$

In the Reynolds Number $Re = \bar{w}\ d/\nu$, the kinematic viscosity ν (m^2/s) depends on the temperature and pressure. In the case of air, we have for the viscosity η ($kg\ s/m^2$) the well-known relation*

$$10^6\ \eta = 1.712\ \sqrt{1 + 0.003665\ t}\ (1 + 0.00080\ t)^2$$

where $\nu = \eta/\rho$ and $\rho = \Psi/g\ \overline{w}$. The temperature is computed from $\overline{T} = p\overline{w}/R\Psi$.

We now have to compute the values of Re and λ. For comparison with previous measurements these values will be presented in the form $1/\sqrt{\lambda}$ and $\log\ (Re\sqrt{\lambda})$ (fig. 10). The continuous straight line corresponds to the previous measurements with water according to the equation

$$1/\sqrt{\lambda} = -\ 0.8 + 2\ \log\ (Re\sqrt{\lambda})\ \text{(equation 4)} \tag{20}$$

Since in the Reynolds Number Re the kinematic viscosity ν changes approximately at the same rate as the velocity, the viscosity remains approximately constant for all velocities. Only a change in the pipe diameter d has any large effect on Re. In spite of a certain degree of scattering of the points, unavoidable with measurements at such high air velocities, it is nevertheless evident that the points lie sufficiently close to the straight line drawn and hence show that equation (20) may also be applied to flows with high air velocities in smooth straight pipes.

4. Velocity distribution.- In addition to the pressure variation along the pipe there was also determined the velocity distribution over the end section of the pipe. For this purpose there were measured the total pressure p_0 with a pressure tube (0.5 mm outer, and 0.3 mm inner diameter), the static pressure p by means of orifices in the pipe wall, and the initial state of the flow on entering the test pipe.

The total pressure p_0 is made up of the static pressure and the dynamic pressure. The latter arises from the flow energy through adiabatic compression and represents the conversion of the pressure drop of a jet escaping freely without friction from a tank, the representation being exact in the case of velocities below that of sound and approximate to a certain degree in the case of velocities above that of sound on account of the compression shock set up in the latter case.

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*See Hutte, 26 edition, vol. 1, p. 354.

Figure 11 shows the velocity distribution for increasing pipe lengths in the case of velocities below that of sound. The velocity w, which is made nondimensional by division through the maximum velocity W, is plotted against y/r where y is the distance from the pipe wall and r the pipe radius. Since the velocity distribution could be measured only at the ends of the pipe, the pipes were gradually cut off to fixed lengths. The figure shows that the velocity distribution becomes more tapering with increasing pipe length and beyond a length $l/d = 36$ does not vary any longer. The same result was obtained in previous tests on water, so that in this respect there is an agreement between the flow of incompressible and compressible fluids. This result can be confirmed by figure 11 only for the subsonic region. For the supersonic region such confirmation is not possible since the supersonic velocity, even at the smallest practically attainable pressure ratios p/p_k, can only occur in very short pipes on account of the compression shock already mentioned. For this reason the measurement of the velocity distribution for supersonic velocities was omitted.

The same figure shows the velocity distributions for subsonic velocities plotted as a function of Re for $l/d > 36$ as well as two velocity distributions taken from the measurements of Nikuradse with water. The test values fit the Nikuradse values with respect to the Reynolds Numbers and show that here, too, there is practical agreement. For the supersonic region the velocity distribution was not measured since the complete distribution could not be obtained with the short pipes required for the supersonic velocities.

5. Compression shock.- The setting up of a compression shock in smooth, straight pipes is explained by the fact that the velocity and density of the gas, with rise in pressure, vary at different rates, the velocity decreasing more rapidly than the density is increasing in the case of velocities above that of sound. The reverse is true for velocities below that of sound. Now the quantity flowing through unit area of pipe cross section is the product of the velocity and the density, and for a cylindrical pipe, is constant. With supersonic flow in cylindrical pipes the pressure rises in the direction of flow and from the property of the gas, just mentioned, at any section of the pipe a condition must occur where the quantity entering the pipe and determined by the nozzle at the pipe entrance, can no

longer flow through for the same flow conditions. Only by a sudden transition of the flow to the subsonic state, with a corresponding decrease in the frictional resistance, can the flow be maintained. This is where the compression shock arises. No reaction of the disturbance caused by the pressure jump can occur on the supersonic flow upstream since the velocity of the flow is here greater than that of the disturbance which, as is known, is propagated with the velocity of sound. The reaction of the subsonic interval shows up only in the position of the compression shock in the pipe.

Figure 12 shows that the position of a compression shock in a smooth, straight pipe travels in the opposite direction to that of the flow as the throttling at the end of the pipe is increased. For a given throttling condition the compression shock stays at a fixed position in the pipe. The flow downstream of the compression shock occurs at below sound velocity and changes with change in the throttling, whereas upstream of the compression shock the velocity is supersonic and entirely independent of the throttling.

The investigations were carried out with the pipe arrangement of figure 3, the velocity measuring apparatus at the end of the pipe being modified into a throttling device.

III. SUMMARY

In the foregoing work the flow conditions in smooth, straight pipes and at high air velocities are investigated. A relation is obtained between the quantity flowing through unit area, the pressure gradient along the pipe, and the pipe length.

There is further determined the friction coefficient λ and compared with previous measurements on incompressible fluids. The result is obtained that for high flow velocities the laws of Nikuradse may be applied to compressible fluids.

The velocity distributions over the cross section of the pipe were measured with a pitot tube and agree essentially with those previously obtained for incompressible fluids.

Finally, there was investigated the behavior of the compression shock in a smooth cylindrical pipe. The compression shock can occur at any position in the pipe, depending on the throttling downstream, and travels upstream with increasing throttling up to the pipe entrance, so that thereafter only subsonic velocities occur in the pipe.

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REFERENCES

1. Prandtl, L.: Zur turbulenton Strömung in Röhren und längs Platten. Ergb. Aero. Vers., Göttingen, IV Lfg., 1932; ders. Z.V.D.I. 77, 1933, p. 105.

Nikuradse, J.: Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren. V.D.I., Forschungsheft 356, Berlin, 1932.

von Karman, Th.: Mechanical Similitude and Turbulence. T.M. No. 611, N.A.C.A., 1931.

2. Zeuner, G.: Technische Thermodynamik. Second edition, p. 250. Leipzig, 1900.

3. Nikuradse, J.: Gesetzmassigkeiten der turbulenten Strömung in glatten Rohren. V.D.I., Forschungsheft 356, Berlin, 1932.

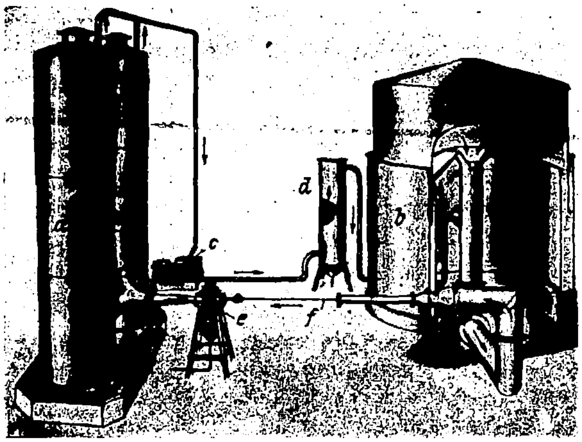
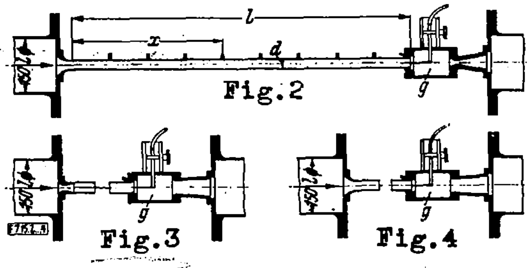


Figure 1.- Test set-up; a,vacuum tanks; b,gas reservoir; c,vacuum pump; d,air drying apparatus; e,shut-off cock; f,test pipe.



Figures 2,3,4,- Test pipe;g,velocity measuring apparatus. Figure 2.-For below-sound velocities.
Figure 3.- For above-sound velocities.
Figure 4.- For sound velocity at end section of pipe.

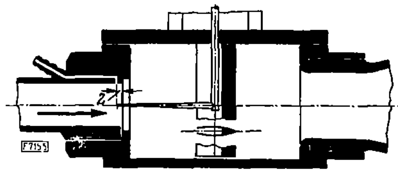


Figure 5.- Velocity measuring apparatus with Pitot tube and covering on Pitot tube stem.

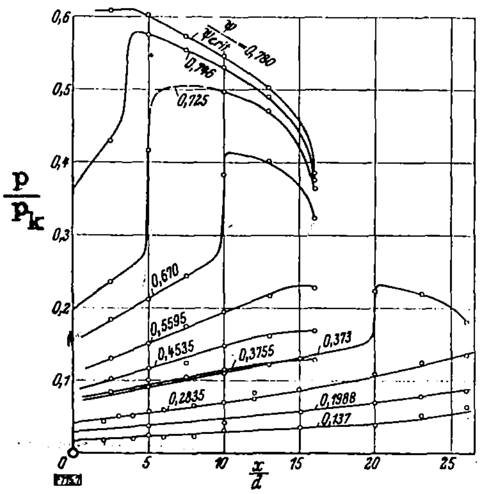


Figure 7.- Pressures p/p_k plotted against lengths x/d for above-sound velocities for 25 mm pipe diameter.

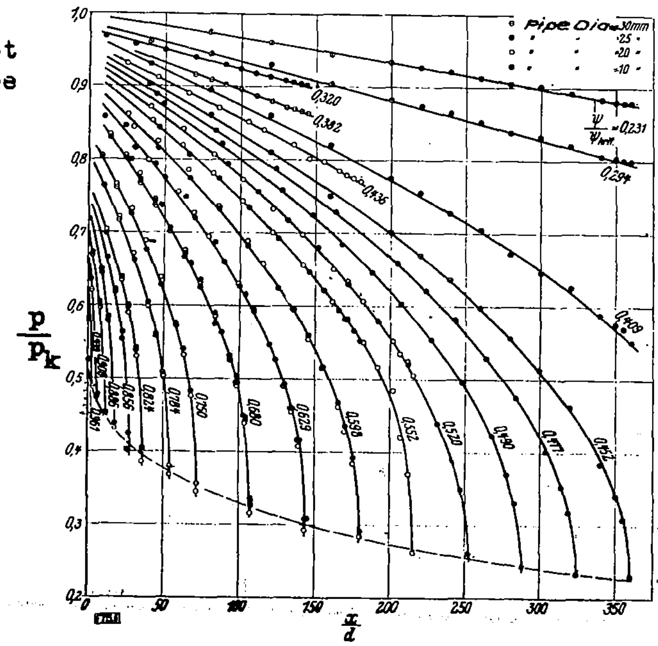


Figure 6.- Pressures p/p_k plotted against lengths x/d for below-sound velocities.

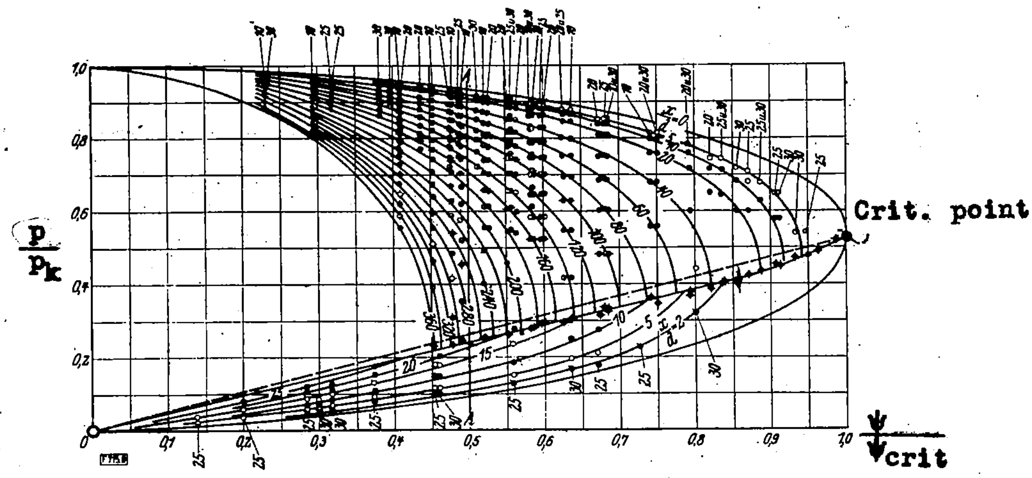


Figure 8.- p/p_k as a function of ψ/ψ_{crit} for above- and below-sound velocities.

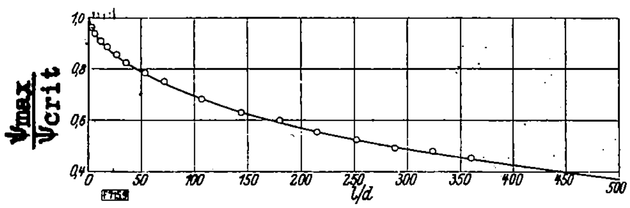


Figure 9.- ψ_{max}/ψ_{crit} as a function of l/d.

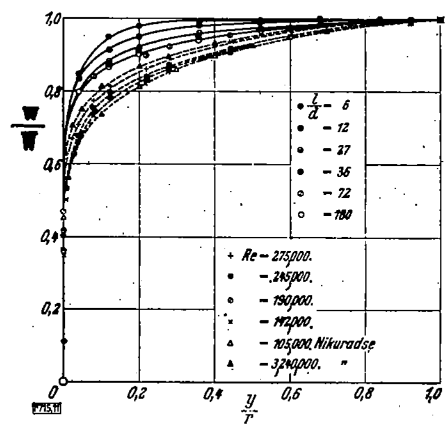


Figure 11.- w/W as a function of y/r for various pipe lengths and Re values.

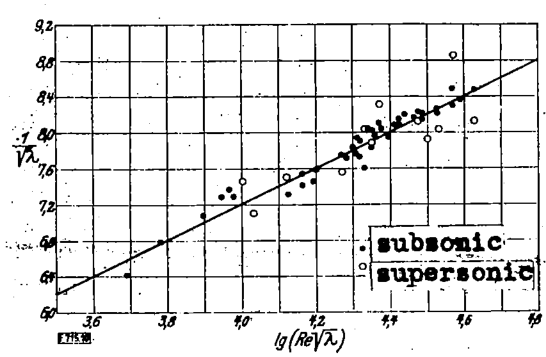


Figure 10.- $1/\sqrt{x}$ as a function of log $(Re\sqrt{\lambda})$.

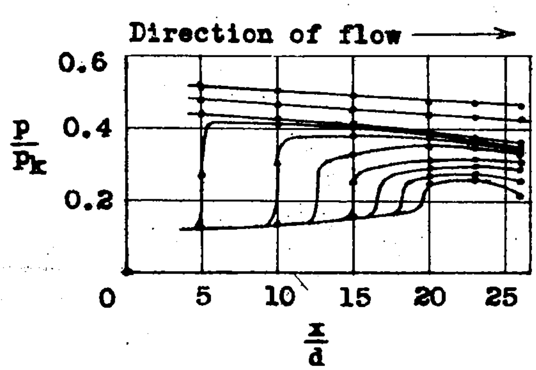


Figure 12.- Travel of compression shock in a smooth cylindrical pipe as throttling is increased.